

# The ideal mixing departure in vector meson physics

L. Epele, H. Fanchiotti, A.G. Grunfeld

Instituto de Física La Plata CONICET, Dto. de Física, Fac. de Cs. Exactas, UNLP, C.C. 67, 1900 La Plata, Argentina

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**Abstract.** In this work we study the departure from the ideal  $\phi$ - $\omega$  mixing angle within the framework of the Nambu–Jona–Lasinio model. In this context we show that the flavor symmetry breaking is unable to produce the shifting of the ideal mixing. We found that a nonet symmetry breaking in the neutral vector sector is necessary to regulate the non-strange content of the  $\phi$  meson. The phenomenology is well described by our proposal.

It is well known that it is not possible to use perturbative expansions of QCD to describe low energy hadronic phenomena. For that reason, a great deal of effective theories, preserving the symmetries of QCD, have been developed to account for the main properties of hadrons. One of the basic aspects common to these schemes is the chiral invariance of the strong interactions in the massless limit. This symmetry is explicitly broken if quarks are massive. In particular, chiral perturbation theory [1] has already demonstrated its ability to describe the low energy hadron phenomena well.

The effective four-fermion Lagrangian proposed by Nambu–Jona–Lasinio (NJL) [2] is an intermediate step between QCD and effective meson theories. From the NJL Lagrangian it is possible to obtain an effective meson Lagrangian after a proper bosonization [3].

The NJL model is a profitable arena to study various phenomena related to symmetry breakdown in hadron physics. In particular,  $\omega$ - $\phi$  mixing largely has been studied from the theoretical and experimental points of view since the beginning of the sixties [4]. In the ideal mixing, the  $\phi$  meson is composed only of strange quarks. However, measurements show that the  $\phi$  meson decays into  $\pi^+\pi^-$ , violating isospin conservation and the OZI rule [5,6].

In the present paper we study the departure from the  $\omega$ - $\phi$  ideal mixing in the NJL model, allowing for a non-strange content in the  $\phi$  meson. Our attention will be particularly focused on the vector meson sector where the phenomenology we are interested in describing takes place. It is important to note that in most of the theoretical models, the  $\rho$  and  $\omega$  mesons are composed by  $u$  and  $d$  light quarks, whereas the quark content of the  $\phi$  meson is purely strange, which does not coincide with the experimental data [7]. Let us remark that some authors have analyzed the departure from ideal mixing within different approaches [8–10].

The NJL model predicts the ideal mixing angle for vector mesons in the process of diagonalization of the neutral vector sector [11,12]. Our purpose is to explore the ori-

gin of the non-ideal  $\phi$ - $\omega$  mixing within this framework in connection with the explicit breakdown of the  $SU(3)$  flavor symmetry to  $SU(2)$  isospin symmetry when assuming  $m_u = m_d \neq m_s$ .

In order to investigate the source of the departure from the ideal mixing, we explore different possibilities. First of all, we present a brief summary of our previous results [12] where we have studied the explicit symmetry breaking in the NJL model. We revisit that scheme specially pointing to the mixing of the  $\phi$ - $\omega$  mesons.

Afterwards, we study the way in which the flavor symmetry breaking, with throughout QCD corrections, modifies the NJL coupling constants and consequently their influence on the  $\phi$ - $\omega$  mixing angle. In the hadron scale we are dealing with, the non-perturbative gluon propagator can be approximated by a universal constant leading to a local NJL Lagrangian [13]. This universal constant is modified neither by chiral symmetry breaking nor by  $SU(3)$  flavor symmetry breaking. However, it is affected if we take into account the QCD vertex corrections when the breakdown of the flavor symmetry is considered. In our present work we show that the ideal mixing angle in the vector sector is modified neither by the explicit flavor symmetry breaking through the mass term of the NJL Lagrangian nor by the breakdown of this symmetry induced by QCD vertex corrections.

Finally and with the objective of investigating the possible source of the ideal mixing departure within the vector meson sector, we modify the coupling constant in the NJL Lagrangian inspired by models for  $\eta$ - $\eta'$  physics [14]. To this end we add a new parameter in order to separate the singlet from the octet vector meson states.

Let us start our analysis with the NJL Lagrangian [2]

$$\mathcal{L} = \bar{q}(i \not{\partial} - \hat{m}_0)q + 2G_1 \left[ \left( \bar{q} \frac{1}{2} \lambda^a q \right)^2 + \left( \bar{q} i \gamma_5 \frac{1}{2} \lambda^a q \right)^2 \right] - 2G_2 \left[ \left( \bar{q} \gamma^\mu \frac{1}{2} \lambda^a q \right)^2 + \left( \bar{q} \gamma^\mu \gamma_5 \frac{1}{2} \lambda^a q \right)^2 \right], \quad (1)$$

where  $q$  denotes the  $N$ -flavor quark spinor,  $\lambda^a$ ,  $a = 0, \dots, N^2 - 1$  are the generators of the  $U(N)$  flavor group (we normalize  $\lambda^0 = \sqrt{2/N} \mathbf{1}$ ) and  $\hat{m}_0$  stands for the current quark mass matrix. The coupling constants  $G_1$  and  $G_2$  as well as the quark masses are introduced as free parameters of the model. In the absence of the mass term, the NJL Lagrangian shows at the quantum level the  $SU(N)_A \otimes SU(N)_V \otimes U(1)_V$  symmetry characteristic of massless QCD.

It is possible to reduce the fermionic degrees of freedom to bosonic ones by the standard bosonization technique. By means of the Stratonovich identity, the vector-vector coupling in (1) can be transformed as

$$-2G_2 \left( \bar{q} \gamma^\mu \frac{1}{2} \lambda^a q \right)^2 \rightarrow -\frac{1}{4G_2} \text{Tr} V_\mu^2 + i \bar{q} \gamma^\mu V_\mu q, \quad (2)$$

where

$$V_\mu \equiv -i \sum_{a=0}^8 V_\mu^a \lambda^a / 2. \quad (3)$$

The spin-1 fields  $V_\mu^a$  can be identified with the usual nonet of vector mesons as in [12], which transform in such a way as to preserve the chiral symmetry of the original NJL Lagrangian (and therefore that of QCD). Notice that the first term in the right hand side of (2) is nothing but a mass term for the vector fields  $V_\mu^a$ ; thus the vector meson masses are governed by the coupling  $G_2$  in the NJL Lagrangian. It can be seen that these masses are degenerate in the limit where the quark masses are degenerate. The quark fields can be integrated out, leading to an effective Lagrangian which only contains bosonic degrees of freedom. This procedure can be carried out by taking into account the generating functional and performing the calculation of the fermion determinant (a detailed analysis can be found in [3]). A similar procedure can be followed for the full NJL Lagrangian (1), leading to the interactions involving scalar, pseudoscalar and axial-vector bosons. In this way, the final effective Lagrangian is written only in terms of spin-0 and spin-1 colorless hadron fields.

In our previous work [12] we have studied the explicit flavor symmetry breaking in the NJL model when  $m_u = m_d \neq m_s$ , obtaining the ideal mixing in the process of diagonalizing the neutral vector meson sector. We have performed the bosonization by carrying out an expansion of the fermion determinant, which gives rise to a set of one-loop Feynman diagrams [15]. The generating functional gives rise to effective kinetic terms for the spin-1 vector mesons via one-loop diagrams, giving the following contribution to the vector meson self-energy:

$$iN_c \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \frac{\not{k} - \not{p} + m_1}{(k-p)^2 - m_1^2} \lambda^a \gamma_\mu \frac{\not{k} + m_2}{k^2 - m_2^2} \lambda^b \gamma_\nu, \quad (4)$$

where  $m_1$  and  $m_2$  are the constituent masses of the quarks entering the loop,  $N_c$  is the number of colors, and the trace acts over the flavor and Dirac indices.

We will take only the leading order in the external momentum  $p$ , which means to evaluate the integral at  $p = 0$  after extracting the relevant kinematical factors. In this

case, this is equivalent to only considering the divergent piece of (4):

$$\Pi_{\mu\nu}^{(V)} = I_2(m_1, m_2) \times \left[ \frac{1}{3} (p_\mu p_\nu - p^2 g_{\mu\nu}) + \frac{1}{2} (m_2 - m_1)^2 \right], \quad (5)$$

where

$$I_2(m_i, m_j) \equiv -i \frac{N_c}{(2\pi)^4} \int d^4 k \frac{1}{(k^2 - m_i^2)(k^2 - m_j^2)}. \quad (6)$$

In order to regularize the divergence we use the proper-time regularization scheme with a cut-off  $\Lambda$ , which will be treated as a free parameter of the model. We obtain

$$I_2(m_i, m_j) = \frac{N_c}{16\pi^2} \times \int_0^1 dx \Gamma \left( 0, \frac{(m_i^2 - m_j^2)x + m_j^2}{\Lambda^2} \right). \quad (7)$$

From (5), the kinetic terms for the vector mesons in the effective Lagrangian are given by

$$\begin{aligned} \mathcal{L}_{kin}^{(V)} = & -\frac{1}{4} \frac{2}{3} I_2(m_u, m_u) \\ & \times [\rho_{\mu\nu} \rho^{\mu\nu} + 2\rho_{\mu\nu}^+ \rho^{-\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu} + \alpha \phi_{\mu\nu} \phi^{\mu\nu} \\ & + 2\beta (K_{\mu\nu}^{*+} K^{*- \mu\nu} + K_{\mu\nu}^{*0} \bar{K}^{*0 \mu\nu})], \end{aligned} \quad (8)$$

where  $V^{\mu\nu} \equiv \partial^\mu V^\nu - \partial^\nu V^\mu$ , and

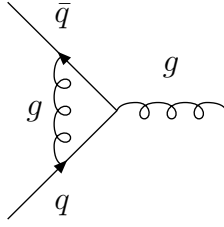
$$\alpha = \frac{I_2(m_s, m_s)}{I_2(m_u, m_u)}, \quad \beta = \frac{I_2(m_u, m_s)}{I_2(m_u, m_u)} \quad (9)$$

parametrize the magnitude of the flavor symmetry breaking.

As was shown in [12],  $SU(3)$  flavor symmetry breaking through the quark mass term in the Lagrangian is not able to produce a departure from the ideal vector meson mixing angle. Then, as mentioned before, another mechanism should be responsible for such effect. Our aim is to perform a further analysis, turning now our attention to the four-fermion interaction in the NJL model.

In QCD, vertex corrections depend on quark propagators and consequently on quarks masses as can be seen in Fig. 1. If quark masses are degenerate, the QCD vertex contributions are the same for all couplings. Nevertheless, if we consider the case of explicit breakdown of the flavor symmetry, the internal lines in Fig. 1 will have different propagators, due to different quark masses, and consequently the contributions coming from vertex corrections at QCD level should modify the four-fermion interaction in the NJL model. We modelled that effect introducing a new flavor symmetry breaking parameter in the strange current coupling constant.

Let us start writing the terms containing the vector current-current interaction in (1). As we are interested in the departure from the  $\phi$ - $\omega$  ideal mixing, we will only



**Fig. 1.** One-loop correction to quark–gluon vertex in QCD

concentrate in the neutral vector current–current terms; the extension to the full Lagrangian is straightforward:

$$-2G_2 \left( \bar{q} \gamma_\mu \frac{\lambda_a}{2} q \right)^2 = -2G_2 \left[ \left( \bar{u} \bar{d} \bar{s} \right) \gamma_\mu \frac{\lambda_a}{2} \begin{pmatrix} u \\ d \\ s \end{pmatrix} \right]^2. \quad (10)$$

Thus, the diagonal terms, driven by  $\lambda_0$ ,  $\lambda_3$ ,  $\lambda_8$ , give us

$$\begin{aligned} J_0 &= \frac{\sqrt{2}}{2\sqrt{3}} \bar{u} \gamma_\mu u + \frac{\sqrt{2}}{2\sqrt{3}} \bar{d} \gamma_\mu d + \frac{\sqrt{2}}{2\sqrt{3}} \bar{s} \gamma_\mu s \\ J_3 &= \frac{1}{2} \bar{u} \gamma_\mu u - \frac{1}{2} \bar{d} \gamma_\mu d \\ J_8 &= \frac{1}{2\sqrt{3}} \bar{u} \gamma_\mu u + \frac{1}{2\sqrt{3}} \bar{d} \gamma_\mu d - \frac{2}{2\sqrt{3}} \bar{s} \gamma_\mu s, \end{aligned} \quad (11)$$

where we have set  $J_a = \bar{q} \gamma_\mu (\lambda_a/2) q$  (for simplicity we have omitted the index  $\mu$ ). Defining  $U = \bar{u} \gamma_\mu u$ ,  $D = \bar{d} \gamma_\mu d$  and  $S = \bar{s} \gamma_\mu s$ , it is easy to see that the sum of these three terms (squared) presents flavor symmetry  $SU(3)$  and it is proportional to  $(U^2 + D^2 + S^2)$ .

Now, if  $m_u = m_d \neq m_s$  the  $S$  current– $S$  current interaction in the NJL Lagrangian is modified in the following way:

$$S^2 \rightarrow (1 + \epsilon)^2 S^2 \quad (12)$$

where the  $\epsilon$  parameter resumes the contributions coming from QCD vertex corrections when considering the explicit breakdown of flavor symmetry. Then, taking into account (12),

$$(U^2 + D^2 + S^2) \rightarrow (U^2 + D^2 + S^2) + \epsilon(2 + \epsilon)S^2. \quad (13)$$

We rewrite the diagonal current–current interactions at  $\mathcal{O}(\epsilon)$ :

$$\begin{aligned} J_0 &= \frac{\sqrt{2}}{2\sqrt{3}} (U + D + S) \rightarrow \\ J'_0 &= \frac{\sqrt{2}}{2\sqrt{3}} (U + D + S + \epsilon S) = \left(1 + \frac{\epsilon}{3}\right) J_0 - \frac{\sqrt{2}}{3} \epsilon J_8, \\ J_3 &= (U - D), \\ J_8 &= \frac{1}{2\sqrt{3}} (U + D - 2S) \rightarrow \\ J'_8 &= \frac{1}{2\sqrt{3}} (U + D - 2S - 2\epsilon S) \\ &= \left(1 + \frac{2\epsilon}{3}\right) J_8 - \frac{\sqrt{2}}{3} \epsilon J_0, \end{aligned} \quad (14)$$

where the primed currents are expressed in terms of a mixture of the non-primed ones, regulated by the  $\epsilon$  parameter.

The NJL Lagrangian vector terms can be expressed in terms of the primed currents, containing the  $\epsilon$  dependence, as follows:

$$\mathcal{L}'_{\text{NJL}}^{(V)} = \bar{q}(i \not{\partial} - \hat{m}_0)q - 2G_2 \sum_{a=0}^8 J_a'^2. \quad (15)$$

In this case, according to the procedure presented in [12], the quark fields can be integrated out, leading to an effective Lagrangian which only contains bosonic degrees of freedom. Taking into account (2), the final Lagrangian can be written in terms of the bosonic fields

$$\begin{aligned} \omega_1 &= \left(1 + \frac{\epsilon}{3}\right) \omega'_1 - \frac{\sqrt{2}}{3} \epsilon \omega'_8, \\ \omega_8 &= \left(1 + \frac{2\epsilon}{3}\right) \omega'_8 - \frac{\sqrt{2}}{3} \epsilon \omega'_1, \end{aligned} \quad (16)$$

warranting that the kinetic energy has the same expression as in (8), with no dependence on the  $\epsilon$  parameter. Note that, from (3), identifying  $V_\mu^a$  as the usual nonet of vector mesons [12],  $V_0$  and  $V_8$  correspond to  $\omega_1$  and  $\omega_8$  respectively.

Then, keeping the first order in the  $\epsilon$  power expansion, the neutral vector mass terms are

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{(V)} &= \frac{1}{8G_2} \\ &\times \left[ \left(1 - \frac{2\epsilon}{3}\right) \omega_1^2 + \rho^2 + \left(1 - \frac{4\epsilon}{3}\right) \omega_8^2 + \frac{4\sqrt{2}}{3} \epsilon \omega_1 \omega_8 \right]. \end{aligned} \quad (17)$$

Both kinetic and mass terms are non-diagonal in the neutral sector. In order to diagonalize simultaneously these two quadratic forms, it is necessary to introduce the following transformation which includes two different angles  $\theta_1, \theta_2$ :

$$\begin{aligned} \omega_8 &= \phi \cos \theta_1 + \omega \sin \theta_2, \\ \omega_1 &= -\phi \sin \theta_1 + \omega \cos \theta_2. \end{aligned} \quad (18)$$

Let us remark that in the literature some authors [8–10] have already introduced two different mixing angles. We found that the kinetic term of (8) is diagonal when the following condition is satisfied:

$$\begin{aligned} (1 + 2\alpha) \tan \theta_2 - (2 + \alpha) \tan \theta_1 \\ + \sqrt{2}(1 - \alpha)(1 - \tan \theta_1 \tan \theta_2) = 0. \end{aligned} \quad (19)$$

On the other hand, replacing (18) in (17) we obtain the following form:

$$\begin{aligned} -2 \left(1 - \frac{2\epsilon}{3}\right) \sin \theta_1 \cos \theta_2 + 2 \left(1 - \frac{4\epsilon}{3}\right) \sin \theta_2 \cos \theta_1 \\ + \frac{4\sqrt{2}}{3} \epsilon (\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) = 0, \end{aligned} \quad (20)$$

which is indeed the mass term diagonalization condition.

In order to estimate the magnitude of  $\epsilon$ , we express the vector meson masses in terms of that parameter. We proceed to the wave function renormalization required by the kinetic terms of (8), redefining the meson fields as in [12]. Considering the mass contributions coming from the divergent one-loop contribution [12], the vector meson masses, expressed in terms of the flavor symmetry breaking parameters are

$$\begin{aligned} m_\rho^2 &= m_\omega^2 = \frac{Z_\rho}{4G_2}, \quad m_{K^*}^2 = \frac{m_\rho^2(1-\epsilon)}{\beta} + \frac{3}{2}(m_s - m_u)^2, \\ m_\phi^2 &= \frac{m_\rho^2(1-2\epsilon)}{\alpha}. \end{aligned} \quad (21)$$

Then, from the experimental value for the  $\phi$  mass [7], we can estimate the value of  $\epsilon$ . In our calculation we have supposed that the flavor symmetry breaking parameters  $\alpha$  and  $\beta$ , are not modified by considering the vertex corrections at QCD level (here we use the phenomenological values for these parameters obtained in [12]). In this way,  $\epsilon$  results in

$$\epsilon \simeq -0.03. \quad (22)$$

Therefore, the coupling constant in the neutral sector (those terms in NJL Lagrangian with  $S$  current-current interactions) and in the charged vector sector are  $0.94G_2$  and  $0.97G_2$  respectively. However, the tiny but non-vanishing vertex corrections that modify the meson masses are not able to shift the ideal mixing angle.

We conclude that the flavor symmetry breaking driven by the parameters  $\alpha$  and  $\beta$  in [12], as well as the parameter which takes into account the QCD vertex corrections  $\epsilon$ , do not lead to any change in the mixing angle in the vector sector. Then, we have to look for another mechanism able to produce a non-vanishing  $u, d$  content of the  $\phi$  meson allowing the decay  $\phi \rightarrow \pi^+\pi^-$  to occur [7].

In the pseudoscalar sector, the presence of the  $U(1)$  anomaly breaks the  $U(3)$  symmetry down to  $SU(3)$ , leading to the mass splitting between the observed  $\eta$  and  $\eta'$  physical states. In the NJL model this effect is reproduced through the 't Hooft determinant interaction [14]. Another way to take into account the anomaly is introducing the  $\eta$ - $\eta'$  mixing angle as a parameter of the model. Inspired by this peculiar physics, we have examined the sensitivity of the mixing angle in the vector sector including a parameter  $\delta$  to force a nonet symmetry breaking. We proceed with the following substitution:

$$-2G_2 J_0'^2 \rightarrow -2G_2' J_0'^2, \quad (23)$$

with  $G_2' \equiv \delta G_2$ . This new parameter, after proper bosonization, can be absorbed in the quadratic term; thus,

$$-2G_2' J_0'^2 \rightarrow \left( \frac{-1}{4G_2'} V_0'^2 + V_0' J_0' \right). \quad (24)$$

Now, keeping the  $\epsilon$  dependence in the mass terms, we analyze the effect that  $\delta$  produces after bosonization. Proceeding as before, let us rewrite the neutral mass terms including the new parameter  $\delta$ :

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{(V)} &= \frac{1}{8G_2} \left[ \frac{1}{\delta} \left( 1 - \frac{2\epsilon}{3} \right) \omega_1^2 + \rho^2 + \left( 1 - \frac{4\epsilon}{3} \right) \omega_8^2 \right. \\ &\quad \left. + \frac{2\sqrt{2}}{3} \epsilon \left( 1 + \frac{1}{\delta} \right) \omega_1 \omega_8 \right]. \end{aligned} \quad (25)$$

Note that the mixing in the quadratic neutral sector term is regulated by the  $\delta$  and  $\epsilon$  parameters. It is easy to see that in the limit  $\delta \rightarrow 1$  we again obtain (17). As above, we have diagonalized both kinetic and quadratic terms by considering

$$\begin{aligned} \omega_8 &= \phi \cos \theta_1 + \omega \sin \theta_2, \\ \omega_1 &= -\phi \sin \theta_1 + \omega \cos \theta_2. \end{aligned} \quad (26)$$

The kinetic energy is diagonal when

$$\begin{aligned} (1 + 2\alpha) \tan \theta_2 - (2 + \alpha) \tan \theta_1 \\ + \sqrt{2}(1 - \alpha)(1 - \tan \theta_1 \tan \theta_2) = 0, \end{aligned} \quad (27)$$

and the mass term if

$$\begin{aligned} \frac{-2}{\delta} \left( 1 - \frac{2\epsilon}{3} \right) \sin \theta_1 \cos \theta_2 + 2 \left( 1 - \frac{4\epsilon}{3} \right) \sin \theta_2 \cos \theta_1 \\ + \frac{2\sqrt{2}}{3} \epsilon \left( 1 + \frac{1}{\delta} \right) (\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) = 0. \end{aligned} \quad (28)$$

It is straightforward to see that the ideal mixing angle does not satisfy simultaneously the conditions (27) and (28) when  $\delta \neq 1$ . Note that the above relations become trivial when  $m_u = m_d = m_s$ . Then,  $\alpha = 1$  and  $\epsilon = 0$  are excluded in the following calculations. For  $\delta = 1$  we trivially obtain (20) again.

Now we evaluate the expressions of the vector meson masses in order to compute the magnitude of  $\delta$  and the two mixing angles. Again, we have renormalized the wave function as in [12]. Then, from the new mass term (25) we have

$$\begin{aligned} m_\rho^2 &= \frac{Z_\rho}{4G_2}, \quad m_{K^*}^2 = \frac{m_\rho^2(1-\epsilon)}{\beta} + \frac{3}{2}(m_s - m_u)^2, \\ m_\omega^2 &= m_\rho^2 \left[ \frac{1}{\delta} \left( 1 - \frac{2\epsilon}{3} \right) \cos^2 \theta_2 + \left( 1 - \frac{4\epsilon}{3} \right) \sin^2 \theta_2 \right. \\ &\quad \left. + \frac{2\sqrt{2}}{3} \epsilon \left( 1 + \frac{1}{\delta} \right) \sin \theta_2 \cos \theta_2 \right], \\ m_\phi^2 &= \frac{m_\rho^2}{\alpha} \left[ \frac{1}{\delta} \left( 1 - \frac{2\epsilon}{3} \right) \sin^2 \theta_1 + \left( 1 - \frac{4\epsilon}{3} \right) \cos^2 \theta_1 \right. \\ &\quad \left. - \frac{2\sqrt{2}}{3} \epsilon \left( 1 + \frac{1}{\delta} \right) \sin \theta_1 \cos \theta_1 \right]. \end{aligned} \quad (29)$$

Experimentally, the mixing angle is near  $38^\circ$  [7] (by  $\sim 3^\circ$  different from the ideal mixing angle); defining  $x$  by

$$\tan \theta_1 = \frac{1}{\sqrt{2}} + x \quad (30)$$

as the shifting in the ideal mixing angle, when using (30) in condition (27), we obtain

$$\tan \theta_2 = \frac{1}{\sqrt{2}} + \frac{x}{\alpha} + \mathcal{O}(x^2). \quad (31)$$

Replacing both  $\tan\theta_1$  and  $\tan\theta_2$  in (28), and keeping terms at  $\mathcal{O}(\delta-1)$  and  $\mathcal{O}(\epsilon)$  (neglecting the ones of  $\mathcal{O}((\delta-1)\epsilon)$ ), for the expression for  $x$  the result is found that

$$x \simeq \frac{1}{\sqrt{2}} \frac{1-1/\delta}{1-1/\alpha}. \quad (32)$$

It is interesting to express the physical states  $\phi_F$  and  $\omega_F$  in terms of the “ideal ones” ( $\phi_I$  and  $\omega_I$ ) when considering a shifting in the ideal mixing angle

$$\begin{aligned} \phi_F &= \phi_I - \frac{1}{\alpha} x \omega_I \\ \omega_F &= \omega_I + x \phi_I. \end{aligned} \quad (33)$$

From the above expressions we can see that the non-strange decays of the  $\phi$  meson are controlled by  $x$ , i.e. the shifting in the  $\theta_1$  mixing angle. Then, from (30) and (32) together with the experimental value for the ideal mixing angle shifting, we can determine the phenomenological value for the parameter  $\delta$  which leads to the following relation between the coupling constant:

$$G'_2/G_2 \simeq 0.953. \quad (34)$$

Though quite small, the difference between the  $G'_2$  and  $G_2$  coupling constants becomes crucial to model the departure from the ideal mixing.

Summing up, we have devoted our phenomenological analysis to a study of the departure from the ideal mixing in the vector meson sector in the framework of the NJL model. We have analyzed different mechanisms.

As a starting point we have revisited our previous results [12] focusing on the  $\phi$ - $\omega$  mixing angle. We show that the explicit flavor symmetry breaking when  $m_u = m_d \neq m_s$  in the NJL Lagrangian, does not lead to a non-strange content of  $\phi$  meson.

As a second possible source of the ideal mixing departure, we have analyzed the effects induced by vertex corrections at QCD level. In this case, we have shown that the effective couplings in NJL Lagrangian associated with the strange current are modified by a factor  $1 + \epsilon$  where the parameter  $\epsilon$ , that has been evaluated in terms of the vector meson masses, accounts for the flavor symmetry breaking in the interaction Lagrangian. Nevertheless, after bosonization, both kinetic and mass terms are still diagonalized by the ideal mixing angle. Consequently, in this framework,  $\phi$  is strictly a strange meson, leading to the conclusion that another mechanism should be responsible for the ideal mixing departure.

As a final step, inspired by the peculiar physics of the neutral pseudoscalar sector, we have forced a nonet

symmetry breaking in the vector sector. For that purpose we have included the  $\delta$  parameter as the way to follow the behavior of the  $\phi$ - $\omega$  angle when the  $U(3)$  symmetry is broken down. In this scheme, after proper bosonization, we have diagonalized simultaneously both kinetic and mass terms requiring now two different non-ideal angles, allowing in this way a non-strange content of the  $\phi$  meson. We have expressed the quark content of the physical  $\phi$  and  $\omega$  mesons in terms of the shifting from the ideal mixing. Finally, we have determined a phenomenological value for the  $U(3)$  symmetry breaking parameter  $\delta$ , in terms of the experimental shifting of the ideal mixing angle and of the flavor symmetry breaking parameters  $\alpha$  and  $\epsilon$ .

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